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REGULAR PAPER

Exploring the feasibility of focusing CW light through a scattering medium into closely spaced twin peaks via numerical solutions of Maxwell's equations

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Here we present a numerical simulation to analyze the effect of scattering on focusing light into closely-spaced twin peaks. The pseudospectral time-domain (PSTD) is implemented to model continuous-wave (CW) light propagation through a scattering medium. Simulations show that CW light can propagate through a scattering medium and focus into closely-spaced twin peaks. CW light of various wavelengths focusing into twin peaks with sub-diffraction spacing is simulated. In advance, light propagation through scattering media of various number densities is simulated to decipher the dependence of CW light focusing phenomenon on the scattering medium. The reported simulations demonstrate the feasibility of focusing CW light into twin peaks with sub-diffraction dimensions. More importantly, based upon numerical solutions of Maxwell's equations, research findings show that the sub-diffraction focusing phenomenon can be achieved with scarce *or* densely-packed scattering media.

1. Introduction

Utilizing optical wavelengths for biomedical applications has attracted much attention, partially due to the non-ionizing nature of light. However, application of optical wavelengths is normally limited to surface applications since light penetration is inhibited by scattering medium. The range of application can be significantly expanded if light can propagate deeper and focus with finer resolution through scattering medium.

Enhancing spatial resolution of light focusing through scattering medium is essential to all optical techniques. The applicability of optical techniques depends upon the spatial resolution of optical wavelengths, which is bounded by the diffraction limit.[1] In vacuum, light propagates freely, however, many media in Nature such as fog, cloud, or biological tissues, appear opaque due to scattering, which limits the penetration depth of light. Owing to the limited penetration depth, optical techniques in medicine such as photodynamic therapy[2] and cosmetic laser surgeries[3] are typically restricted to superficial applications. If optical wavelengths can propagate deeper through scattering medium, the applicability of optical techniques can be extended significantly beyond superficial applications.

Much effort has been devoted to increasing the penetration depth of optical wavelengths through scattering medium.[4–7] Derived based upon Dorokhov's random-matrix theory,[8] the transmission matrix model is used to analyze light propagation through scattering medium.[6,9,10] The concept of transmission eigenchannels is typically employed to analyze light propagation through scattering medium; the open channels allow coherent input beam to transmit through a strong scattering medium with order unity efficiency.[5,11–13] This theoretical framework has become the widely used to analyze light transmission through scattering medium.

Furthermore, much experimental progress has been reported. In 2007, Lerosey et al. have reported focusing microwave with a *random distribution* of scatterers achieving resolution beyond the conventional diffraction limit.[14] Research findings suggest that, though opacity of scattering medium hinders light penetration, presence of a scattering medium can be beneficial to enhancing light penetration.[5,7] In recent years, more experimental progress has been made as reported by various groups, including the time-reversal of ultrasonically encoded light (TRUE) technique,[15] the time reversal of variance-encoded light (TROVE) technique,[16] and other achievement reported by various groups.[5,17,18] By exploiting the wave characteristics of light, these reported techniques have a common goal: to enhance penetration of light propagation through turbid media.

Overcoming the diffraction limit to achieve sub-diffraction resolution has been the ultimate goal of optics. In 2000, Pendry[19] introduced a perfect lens with negative refractive index which suggested the possibility of exceeding the diffraction limit. The possibility to achieve spatial resolution beyond the diffraction limit is very encouraging; it enables new possibilities of employing optical techniques to problems involving opaque scattering medium such as biological tissue structures. Recently reported research suggest that the presence of a scattering medium was thought to play a crucial role in the sub-diffraction focusing phenomenon.[5,7,14] In this paper, based upon numerical solutions of Maxwell's equations, we model light focusing into closely spaced twin peaks through a scattering medium. We investigate how scattering contributes to the sub-diffraction focusing phenomenon. Specifically, we model light propagation through scattering medium of various number densities. Our goal is to determine how scattering contributes to the sub-diffraction focusing phenomenon.

2. Research design

To analyze the spatial resolution of light focusing through scattering medium, we report a two-dimensional (2D) simulation using the pseudospectral time-domain (PSTD) algorithm[20] and model CW light propagation through a scattering medium and focus into closely-spaced twin peaks. The scattering medium consists of closely-packed dielectric cylinders. A schematic of the simulation is shown in Fig. 1.

Light scattering can be simulated by various methods, such as the coupled-dipole model;[4] here we report a 2D simulation of CW light propagation through a macroscopic scattering medium by means of numerical solutions of Maxwell's

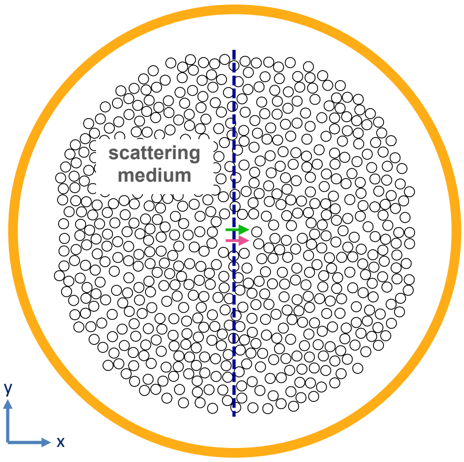

Fig. 1. (Color online) Schematics of analyzing the spatial resolution of CW light focusing through a scattering medium. The CW light from a fully-surrounding wavefront (yellow shaded *OPC region*) focuses into twin peaks at the center. The wavefront is generated by phase-conjugating the amplitude and phase recorded beforehand for light propagating outwards from the center through the scattering medium. The focused twin-peak profiles are measured (y) along the blue dashed line. The cross-sectional light profile of the focused twin peaks is recorded along blue dashed line.

equations via the PSTD algorithm. With economic computational memory usage, the PSTD algorithm is advantageous for modeling macroscopic light scattering problems. A central difference scheme is employed to calculate the temporal derivatives, whereas the discrete Fourier transform is used to calculate the spatial derivatives:

$$\left\{ \frac{\partial E}{\partial x}\bigg|_i \right\} = \mathbf{F}^{-1}\{jk_x \cdot \mathbf{F}\{E_i\}\}, \tag{1}$$

where $\boldsymbol{E}$ is the electric vector field, k_x is the wave number, and the Fourier transform and inverse Fourier transform are denoted by $\mathbf{F}$ and $\mathbf{F}^{-1}$. With a two grid-points per wavelength coarse grid, the PSTD algorithm enables achieving similar accuracy as the finite-difference time-domain (FDTD) technique, which requires 20 grid-points per wavelength. Thus, the coarse requirement of computer memory enables modeling a macroscopic complex light scattering phenomenon. The simulations are of TM waves, where the electric field (E_z) is in the z-direction. The PSTD simulation is a grid-based simulation that can model arbitrary geometry; the accuracy increases with increased grid resolution. With a temporal resolution $\Delta t = 0.05$ fs and spatial resolution of 0.33 µm, we simulate CW light of various wavelengths propagating through a scattering medium (consisting of N randomly-positioned, non-absorbing, dielectric cylinders) with dimensions in a 600-µm-by-600-µm region. Furthermore, to model an isolated system, an absorbing boundary condition[21] is implemented to absorb all outgoing waves.

3. Simulation results

To analyze how scattering affects the spatial resolution of CW light focusing, light propagation through an absorption-less scattering medium is simulated and compared with the ideal case. By employing the PSTD simulation technique for a 2D region of (600 µm)^2, a ring-shaped, CW wavefront is simulated; the CW wavefront propagates through a scattering medium and focus into twin peaks at the center, as shown in

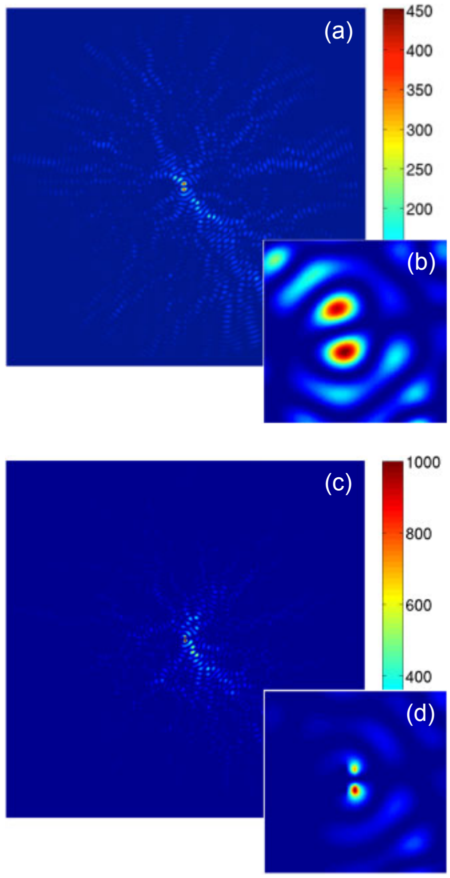

Fig. 2. (Color online) Modeling light ($\lambda = 16$ µm) propagating through a scattering medium and focus into twin peaks (spacing $s = 4$ µm). The 360-µm-diameter scattering medium consists of $N = 1600$ 6-µm-diameter dielectric ($n = 1.2$) cylinders. (a) Light propagates through the scattering medium and converges into blurred twin peaks. (c) By eliminating the outgoing light with a soft sink, the impinging light focuses into fine twin peaks. (20-µm)^2-zoomed-in views of (a) and (c) are shown in (b) and (d), respectively.

Fig. 1. The scattering medium consists of N randomly positioned, 6-µm-diameter dielectric ($n = 1.2$) cylinders clustered in a 290-µm-radius region. The specific phase and amplitude to focus light through the scattering medium is obtained by phase-conjugating the amplitude and phase recorded in the yellow-shaded region (Fig. 1) beforehand for light propagating outwards through the scattering medium from the center. Simulation shows that with the phase-conjugated amplitude and phase, CW light can propagate through the scattering medium and focus into twin peaks.

To determine the dependence of the sub-diffraction focusing phenomenon on scattering, we model CW light propagating through an *absorption-less*, scattering medium and focusing into closely-spaced twin peaks. The cross-sectional electric field E_z profile of light at the target position is compared. Without absorption, light focusing at the center continues to propagate outwards. The incoming light and outgoing light interferes resulting in a blurred twin-peak pattern [Figs. 2(a) and 2(b)]. This phenomenon is similar to the phenomenon of water droplets falling onto a round bucket of water. The impact of a water droplet upon the water surface forms a circular wave diverging outwards. As the outgoing circular wave is reflected by the boundary of the round bucket, the wavefront converges into a point at the center, then, continues to diverge outwards. Normally the

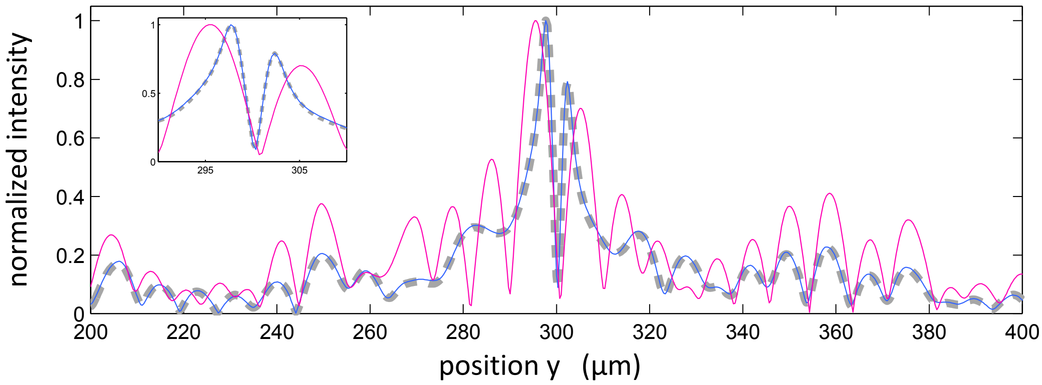

Fig. 3. (Color online) Comparison of the twin-peak profiles. The focused light intensity profile vs position (y) along blue dashed line (Fig. 1) is depicted. (gray dashed line): the ideal twin-peak profile, (pink line): phase-conjugated light focused into twin peaks, broadened by the interference of incoming and outgoing light. (blue line): a soft sink to eliminate the outgoing light component; without the outgoing light, the incoming phase-conjugated light reconstructs the ideal twin-peak profile.

focused point is not visible, as it is blurred by the interference of the incoming wave and outgoing wave, similar to the broadened peak (pink line) as shown in Figs. 2 and 3.

To analyze the sub-diffraction focusing phenomenon of the incoming light, the outgoing light needs to be eliminated. By exploiting the linear nature of the Maxwell's equations, a *soft sink*[22] is employed to eliminate the outgoing light component. Without the outgoing light component, the incoming light focuses into twin peaks [Figs. 2(c) and 2(d)] that matches the original light source profile, similar to time reversal.[23] The total energy of the focused light profiles is normalized to facilitate comparison; the cross-sectional profile of the electric field E_z along the blue dashed line (Fig. 1) at the center of the simulation is recorded. By comparing the focused light profile and the original light source profile, the spatial resolution of CW light focusing can be quantified.

The profile of focused light is depicted and shown in Fig. 3; (pink line): the incoming light and outgoing light interferes resulting in blurred twin peaks with interference-like oscillations [Fig. 2(b)]; (blue line): by implementing a numerical soft sink to eliminate the outgoing light component, the incoming light alone focuses into twin peaks [Fig. 2(d)] that matches the original twin-peak light source profile (gray dashed line). As shown in Fig. 3, without the interference of the outgoing light component, incoming light focusing into closely spaced twin peaks can be quantitatively analyzed.

Here we model the sub-diffraction focusing phenomenon that has recently attracted much attention.[5,14,24,25] We simulate focusing CW light of wavelength λ into 4-μm-apart twin peaks, as shown in Fig. 4. The 180-μm-radius scattering medium consists of 1600 randomly positioned, 6-μm-diameter dielectric ($n = 1.2$) cylinders. With phase-conjugated amplitude and phase, a fully-surrounding 290-μm-radius wavefront (yellow shaded region of Fig. 1) converging on the scattering medium is launched. CW light with wavelength λ focusing into closely-spaced twin peaks are modeled; as shown in Fig. 4: from bottom to top: $\lambda = 1$, 2, 4, 8, 16, 64, and 512 μm, respectively. The focused twin-peak profile (pink line) is compared to the original twin-peak profile (gray dashed line); the RMS error is: 1.195, 0.286, 0.171, 0.174, 0.123, 0.030, and 0.002, respectively. The minute error

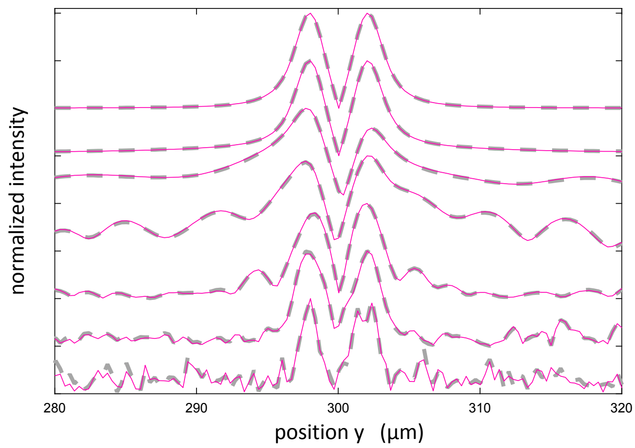

Fig. 4. (Color online) Simulation of CW light of various wavelengths λ propagates through a scattering medium and focuses into twin peaks (4-μm apart). The focused light intensity profiles vs position (y) along the blue dashed line (Fig. 1) are depicted. From bottom to top: $\lambda = 1$, 2, 4, 8, 16, 64, and 512 μm, respectively. The incoming component of CW light focuses into twin-peak (pink line) consistent with the ideal twin-peak profile (gray dashed line) for various wavelength λ.

decreases with increased wavelength, suggesting that it is due to discretization of the simulation. For longer wavelengths, more grid points are used to resolve each wavelength, the simulation is more accurate with higher spatial resolution, thus less stair-casing error. Hence, light focusing into the original twin-peak profile is not dependent on the wavelength (Fig. 4).

Next, to determine whether the number density of the scattering medium plays a crucial role in the sub-diffraction focusing phenomenon, we vary the number of dielectric cylinders of the scattering medium, as shown in Fig. 5. CW light of wavelength $\lambda = 16$ μm focusing through a scattering medium is simulated; within a region of 180-μm-radius, the scattering medium consists of N randomly positioned, 6-μm-diameter dielectric ($n = 1.2$) cylinders. The twin-peak profiles of light focusing through scattering media of various number densities are compared. In Fig. 5, from bottom to top: $N = 0$ (vacuum), 400, 600, 1000, 1200, and 1600. (The morphological disorder[26] corresponding to $N = 1600$ is 0.0757.) As shown in each pair of curves, the focused twin-peak profile (blue

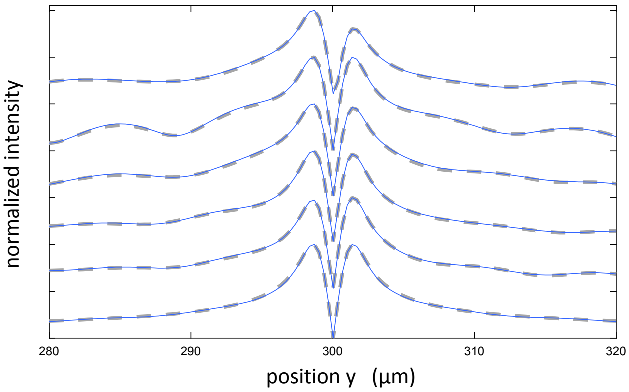

Fig. 5. (Color online) Simulation of CW light ($\lambda = 16\,\mu$m) propagate through a scattering medium and focus into twin peaks (4-μm apart). The focused light intensity profiles vs position (y) along the blue dashed line (Fig. 1) are depicted. From bottom to top: $N = 0$ (*vacuum*), 400, 600, 1000, 1200, and 1600, respectively. The incoming component of CW light focuses into twin-peak (blue line) that matches the ideal twin-peak profile (gray dashed line); the sub-diffraction focusing can be achieved through scattering medium of various number densities (see Video 1 in the online supplementary data at http://stacks.iop.org/JJAP/57/042001/mmedia).

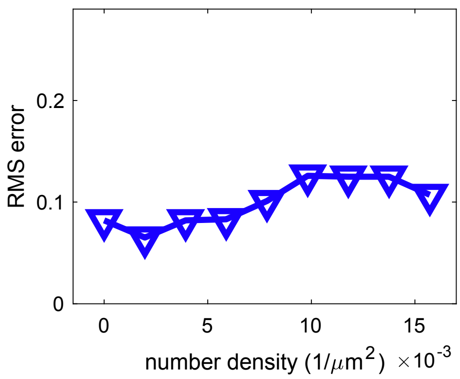

Fig. 6. (Color online) The RMS error of CW light focusing into twin peaks vs number density of the scattering medium. The accuracy of the sub-diffraction focusing phenomenon is not crucially dependent on the number density.

line) is compared to the ideal twin-peak profile (gray dashed line). Without the interference of outgoing light, the incoming light focuses into twin-peak profiles with sub-diffraction resolution for scarce *or* densely-packed scattering medium.

Furthermore, a quantitative analysis of the focused twin-peak profiles (Fig. 5) of light focusing through scattering medium of various number densities (blue line) is compared to the original twin-peak profile (gray dashed line). The RMS error is 0.082, 0.082, 0.083, 0.126, 0.125, and 0.107, respectively, as shown in Fig. 6. Notice that even for $N = 0$, the error is not zero; this is due to the discretization of the grid-based PSTD algorithm. The minute error independent of N suggests that the sub-diffraction phenomenon is not dependent upon the number density of the scattering medium.

The accuracy of the simulation can be ascertained by the numerical convergence analyses of Fig. 7. The error of the simulation decreases with longer simulation time (approaching the CW steady state), as shown in Fig. 7(a). To model phase conjugation of light in the far field, a large enough simulation region is required to ensure outgoing light reaches the OPC region with near-normal incidence. As shown in

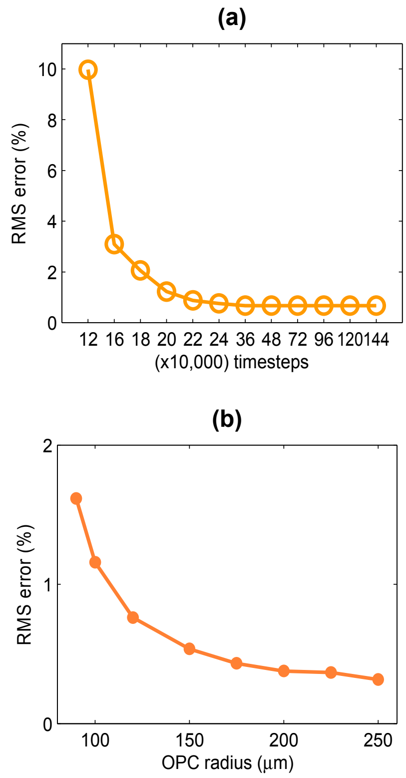

Fig. 7. (Color online) Simulation convergence analyses. The accuracy increases with: (a) increased number of simulation time-steps (approaching the CW steady state), and, (b) increased distance between the OPC region and the scattering medium, where outgoing light reaches the OPC region become closer to normal incidence.

Fig. 7(b), convergence analysis shows that the error decreases monotonically with larger OPC radius. Together, the temporal convergence [Fig. 7(a)] and spatial convergence [Fig. 7(b)] demonstrates the accuracy and robustness of the reported simulations.

4. Summary

Presence of a scattering medium was thought to be crucial for the sub-diffraction focusing phenomenon.[5,7,14] Here we present a numerical simulation to analyze the effect of scattering on the sub-diffraction focusing phenomenon. We simulate focusing of CW light through a scattering medium into closely-spaced twin peaks with sub-diffraction resolution (Fig. 4). By varying the number density of the scattering medium, our goal is to decipher the contribution of scattering on the sub-diffraction focusing phenomenon. Simulation results show that sub-diffraction focusing can be achieved for scattering media of various number densities (Fig. 6). Based upon numerical solutions of Maxwell's equations, the reported simulations demonstrate the feasibility of focusing far-field, CW light into closely-spaced twin peaks with sub-diffraction resolution; specific simulation results show that sub-diffraction focusing phenomenon is *not* dependent the scattering medium.

Acknowledgments

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